1 Introduction

This galois package introduces two-dimensional notations for Galois connections.

2 Detailed explanations on Galois connections

If \((L, \leq)\) and \((M, \sqsubseteq)\) are posets, \(\alpha \in L \mapsto M, \gamma \in M \mapsto L\) and \(\forall x \in L, y \in M: \alpha(x) \sqsubseteq y \iff x \leq \gamma(y)\) then the pair \((\alpha, \gamma)\) is a Galois connection, written \(\galois{\alpha}{\gamma}: (L, \leq) \xrightarrow{\alpha, \gamma} (M, \sqsubseteq)\):

In a Galois connection, \(\alpha\) is onto if and only if \(\gamma\) is one-to-one if and only if \(\alpha \circ \gamma = 1\) (where \(\circ\) is the functional composition and \(1\) the identity), written \(\galoiS{\alpha}{\gamma}: (L, \leq) \xrightarrow{\alpha, \gamma} (M, \sqsubseteq)\):

\(\alpha\) is one-to-one if and only if \(\gamma\) is onto if and only if \(\gamma \circ \alpha = 1\), written \(\Galois{\alpha}{\gamma}: (L, \leq) \xleftarrow{\alpha, \gamma} (M, \sqsubseteq)\):

For a bijection, we write \(\GaloiS{\alpha}{\gamma}: (L, \leq) \xleftarrow{\alpha, \gamma} (M, \sqsubseteq)\):

The surjection on the quotient of \(M\) by the equivalence relation \(x \equiv y\) defined by \(\gamma(x) = \gamma(y)\) is denoted \(\galoiSr{\alpha}{\gamma}: (L, \leq) \xrightarrow{\alpha, \gamma} (N, \preceq)\):

The composition of Galois connections:

\(\comp\) is a Galois connection (the composition \(\circ\) of functions is \(\comp\):

\(\galois{\alpha}{\gamma} \circ \galois{\beta}{\delta} = \galois{\alpha \circ \beta}{\gamma \circ \delta}\)
\[(L, \leq) \xmapsto{\gamma_1 \circ \gamma_2}{\alpha_2 \circ \alpha_1} (N, \leq)\]

Galois connections \((L, \leq) \xmapsto{\alpha} (M, \sqsubseteq)\) can be lifted from sets of properties to sets of monotone functions:

\[(L \xmapsto{m} L, \leq) \xmapsto{\lambda \phi \circ \alpha \circ \gamma \circ \phi \circ \alpha \circ \gamma}{\lambda \phi \circ \alpha \circ \gamma} (M \xmapsto{m} M, \sqsubseteq)\]

where the ordering on functions is pointwise that is \(\varphi \leq \phi\) if and only if \(\forall x : \varphi(x) \leq \phi(x)\). Observe that the length of the arrows stretches automatically to the appropriate width.

### 3 Package options

**color** The `color` option is required for colored Galois connections is in

```
\galois{red}\{\alpha}\{\gamma}\xrightarrow{\gamma}{\alpha},
\Galois{\alpha}\{blue\}\{\gamma\} \xrightarrow{\gamma}{\alpha},
\Galois\{red\}\{\alpha\}\{blue\}\{\gamma\} \xrightarrow{\gamma}{\alpha}, or
\comp\{red\}
```

Without 'color' option, these colors are ignored.

\@GALOIScolor \@GALOIScolor is \texttt{\color} with the `color` option and later defined as \texttt{\relax} in absence of `color` option.

1 \DeclareOption{color}{%
2 \def\@GALOIScolor{\color}
3 \ProcessOptions

### 4 Style parameters

You can use Galois connections in any size (footnotes, transparencies, etc.) : tiny \(L \xmapsto{\gamma}{\alpha} M\), scriptsize \(L \xmapsto{\gamma}{\alpha} M\), footnotesize \(L \xmapsto{\gamma}{\alpha} M\), small \(L \xmapsto{\gamma}{\alpha} M\), normalsize \(L \xmapsto{\gamma}{\alpha} M\), large \(L \xmapsto{\gamma}{\alpha} M\), LARGE \(L \xmapsto{\gamma}{\alpha} M\), huge \(L \xmapsto{\gamma}{\alpha} M\), Huge \(L \xmapsto{\gamma}{\alpha} M\). Observe that in \(\xrightarrow{\gamma}{\alpha}\) the width of arrows and height of enclosing box are automatically adjusted according to the size of \(\alpha\) and \(\gamma\). You can adjust the following parameters:

\GaloisStyle : style of upper and lower tags (\scriptstyle by default);
\GaloisArrowThickness

\GaloisArrowThickness : thickness of the arrow stems ¹; (0.1ex by default);

\GaloisArrowsSep

\GaloisArrowsSep : distance between the arrows (0.2ex by default);

\GaloisArrowTagSep

\GaloisArrowTagSep : distance between arrows and tags (0.5ex by default).

For example with:
\begin{verbatim}
\renewcommand{\GaloisArrowsSep}{1\text{cm}}
\renewcommand{\GaloisArrowTagSep}{0\text{pt}}
\end{verbatim}
we get
\begin{verbatim}
−−−−−−−−−−−−−−−−−→−→
\end{verbatim}
while with:
\begin{verbatim}
\renewcommand{\GaloisArrowsSep}{0\text{pt}}
\renewcommand{\GaloisArrowTagSep}{5\text{mm}}
\end{verbatim}
we get
\begin{verbatim}
−−−−−−−−−−−−−−−−−→−→←←−−−−−−−−−−−−−−−−−−
\end{verbatim}
and with
\begin{verbatim}
\renewcommand{\GaloisArrowsSep}{0\text{pt}}
\renewcommand{\GaloisArrowTagSep}{0\text{pt}}
\end{verbatim}
we get
\begin{verbatim}
−−−−−−−−−−−−−−−−−→−→←←−−−−−−−−−−−−−−−−−−
\end{verbatim}

5 Implementation

4 ⟨∗package⟩

Require \texttt{color} package for \texttt{color} option else coloring is ignored.
5 \ifx@GALOIScolor\undefined
6 \def@GALOIScolor#1\relax\%
7 \else

¹ stem is “tige” in french.
Define functional composition $f \circ g(x) = f(g(x))$ (if not already defined e.g. as in \texttt{mathtime.sty}). \comp will draw in color (black by default).

\ifdefined{comp}{% Scan the optional color argument
\newcommand{\comp}{\@ifnextchar[{{@GALOIScomp}{@GALOIScomp[black]}}}% Defined the colored functional composition \@GALOIScomp[\color]
\def{@GALOIScomp}[#1]{\mathchoice
\mathrel{\raisebox{0.2ex}{\@GALOIScolor{#1}\scriptstyle\circ}}%}{\mathrel{\raisebox{0.2ex}{\@GALOIScolor{#1}\scriptstyle\circ}}%}{\mathrel{\raisebox{0.1ex}{\@GALOIScolor{#1}\scriptscriptstyle\circ}}%}{\mathrel{\raisebox{0.1ex}{\@GALOIScolor{#1}\scriptscriptstyle\circ}}}}%
}{%}

Style commands:
\begin{itemize}
\item \texttt{GaloisStyle} Style of $a$ and $b$ in $\frac{b}{a}$, $\frac{b}{a}$, $\frac{b}{a}$ or $\frac{b}{a}$.
\item \texttt{GaloisArrowThickness} Thickness of the arrow stems (0.1ex by default):
\item \texttt{GaloisArrowsSep} Distance between the lower and upper arrows (0.2ex by default):
\item \texttt{GaloisArrowTagSep} Distance between the lower arrow and the top of $a$ and the top-arrow and the bottom of $b$ (0.5ex by default)
\end{itemize}

\begin{itemize}
\item \texttt{GALOISalphadepth} \texttt{GALOISalphaheight} \texttt{GALOISgammadepth} \texttt{GALOISwidth} \texttt{GALOISheight} \texttt{GALOISdepth} \texttt{GALOIStotalheight} \texttt{GALOISGap} \texttt{GALOISalphaarrowwidth} \texttt{GALOISalphaarrowhalfheight} \texttt{GALOISgammaarrowwidth} \texttt{GALOISgammaarrowhalfheight}
\end{itemize}

\texttt{Galois@put(\texttt{x,y-d})\{\texttt{text}\}} puts text at coordinates $(x, y - d)$, in a box of size $0pt \times 0pt$:
\begin{verbatim}
def\Galois@put(#1,#2-#3)#4{\rlap{\smash{\hskip#1\setlength{\@tempdimc}{#2-#3}\raisebox{\@tempdimc}{#4}}}}%
\end{verbatim}

\texttt{GALOISrightarrowfill} \texttt{GALOISrightarrowfill(\texttt{rightarrow})}, see \TeX{}book p. 357.
Stacking $a$, the arrows and $g$ in $\frac{g}{a}$:

\begin{verbatim}
\GALOIS \GALOIS\leftarrowfill \leftarrowfill \{leftarrow\}, see \TeXbook p. 357.
\def\@GALOISleftarrowfill#1{$\m@th \mathord{#1} \mkern-7mu \cleaders\hbox{$\mkern-2mu \smash- \mkern-2mu$}\hfill \mkern-7mu \smash-$}\%

\GALOIS \GALOIS\ca\ca\{-->\}<--\}{a}{g}\ constructs $\frac{g}{a}$. \GALOIS\ca\ca\{-->\}<--\}{--\--\--\--}{a}{g}\ and \GALOIS\ca\ca\{-->\}<--\}{--\--\--\--}{a}{g}\ add colors colora for the $a$-arrow and colorg for their arrow.
\def\@GALOIS#1#2\{\@ifnextchar[\{\@GALOISca{#1}{#2}\}\{\@GALOISca{#1}{#2}\[black]\}\%
\def\@GALOISca#1#2\[#3\]#4\{\@ifnextchar[\{\@GALOIScacg{#1}{#2}\[#3\]{#4}\}\{\@GALOIScacg{#1}{#2}\[#3\]{#4}\[black]\}\%
\def\@GALOIScacg#1#2\[#3\]#4\[#5\]#6\{\@GALOISrightarrow\{\@GALOISleftarrow\{\@GALOIScolor{#3}\@GALOIScolor{#5}\GaloisStyle#4\#6\}\%
\end{verbatim}

\GALOIS@put(0pt,0pt-\@GALOISalphaarrowhalfheight){\@GALOISrightarrow}
\GALOIS@put(0pt,\@GALOISGap-\@GALOISalphaarrowhalfheight){\@GALOISleftarrow}
\setlength{\@GALOISalphatag}{\@GALOISwidth-\@GALOISalphadepth-\@GALOISalphatagwidth}
\settowidth{\@GALOISalphatag}{\@GALOISalphatag}
\divide\@GALOISalphatag by 2
\GALOIS@put(\@GALOISalphatag,0pt-\@GALOISalphaarrowhalfheight){\@GALOISleftarrow}
\GALOIS@put(0pt,\@GALOISGap-\@GALOISalphaarrowhalfheight){\@GALOISrightarrow}
\addtolength{\@GALOISGap}{-\@GALOISAlphaArrowSep}
\settoheight{\@GALOISalphatag}{\@GALOISalphatag}
\addtolength{\@GALOISalphatagwidth}{\@GALOISalphatagwidth}
The text contains definitions and commands for creating mathematical symbols and arrows. It includes the following symbols and commands:

- \addtolength{\@tempdimb}{\GaloisArrowTagSep}\addtolength{\@tempdimb}{\@GALOISGap}\addtolength{\@tempdimb}{\@GALOISgammadepth}\Galois@put(\@tempdima,\@tempdimb-\@GALOISdepth){\@GALOISgammatag}\rule[-\@GALOISdepth]{0pt}{\@GALOIStotalheight}\hspace*{\@GALOISwidth}

- The \galois\{a\}{g} command is defined to represent a function $a \rightarrow g$.

- \galoiS\{a\}{g} represents a function $a \mapsto g$ (onto).

- \galoiSr\{a\}{g} represents a function $a \equiv g$.

- \Galois\{a\}{g} represents a bijection $a \mapsto g$.

The index lists various symbols and commands used in the document, along with their page numbers and line numbers where they are defined or used.
Change History

v0.00
  General: Initial version for LaTeX 2.09 ................................. 1

v1.00
  General: Adapted to LaTeX2e .............................. 1

v1.01
  General: Check "\comp" is not already defined (as in \math-
time.sty) ................................................... 1

v1.02
  General: Rewriting of extendable arrows using the \TeX\ book left/right arrowfill p. 357.
  Added and renamed style parameters ("\GaloisSep" now \GaloisArrowTagSep") .......................... 1

v1.03
  General: Renamed internal macros to avoid interferences with
  other packages \GaloisArrowTagSep
  \GaloisRightarrowfill \rightarrow \@GALOISrightarrowfill
  \GaloisLeftarrowfill \rightarrow \@GALOISleftarrowfill

  .................................................. 1

v1.04
  General: \LaTeX\ package file documentation ...................... 1

v1.05
  General: Added color option ............................... 1